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Optimal Design of Grid-Stiffened Composite Panels Using Global and Local Buckling Analyses

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A design strategy for optimal design of composite grid-stiffened panels subjected to global and local buckling constraints is developed using a discrete optimizer. An improved smeared stiffener theory is used for the global buckling analysis. Local buckling of skin segments is assessed using a Rayleigh–Ritz method that accounts for material anisotropy and transverse shear flexibility. The local buckling of stiffener segments is also assessed. Design variables are the axial and transverse stiffener spacing, stiffener height and thickness, skin laminate, and stiffening configuration, where the stiffening configuration is herein defined as a design variable that indicates the combination of axial, transverse, and diagonal stiffeners in the stiffened panel. The design optimization process is adapted to identify the lightest-weight stiffening configuration and stiffener spacing for grid-stiffened composite panels given the overall panel dimensions, in-plane design loads, material properties, and boundary conditions of the grid-stiffened panel.

Nomenclature

a, b	= axial and transverse stiffener spacing, respectively
$F(\mathbf{X}, r_i)$	= modified objective function
h	= stiffener height
ICON	= design variable for stiffening configuration
i	= generation or iteration cycle in the optimization procedure
LAMI	= design variable for stacking sequence of skin laminate
M	= population size
N_c	= number of design constraints
N_d	= number of design variables
Q	= normalizing constant
r_i	= penalty parameter
$r_i \sum_j^{N_c} [\lvert g_j(\mathbf{X})\rvert + g_j(\mathbf{X})]^2$	= penalty function
t	= skin laminate thickness
t_s	= stiffener thickness
$W(\mathbf{X})$	= weight of panel per unit area
w_s	= volume of skin in unit cell, abt
w_1	= volume of axial stiffeners in unit cell, $2hat_s$
w_2	= volume of transverse stiffeners in unit cell, $2hbt_s$
w_3	= volume of diagonal stiffeners in unit cell, $2ht_s\sqrt{a^2 + b^2}$
$\mathbf{X}$	= design variable vector
λ_G	= global buckling load factor
λ_{sk}	= buckling load factor for skin segment
λ_1, λ_2, λ_3	= buckling load factor for axial, transverse, and diagonal stiffener segments
ρ	= mass density of material

Introduction

AN aircraft in flight is subjected to air loads associated with all flight conditions including maneuver and gust conditions. These external loads are resisted by the structure, and an internal load distribution is established based on the structural layout and external loads. These internal loads, which depend on their location in the aircraft structure, may cause either overall panel buckling of stiffened panels, buckling of the skin between stiffeners, or stiffener crippling. Hence, an efficient and accurate method for developing a buckling-resistant design of general stiffened composite panels is needed to identify the most effective grid-stiffened geometries for structural panels subjected to combined in-plane loading conditions at different locations in fuselage and wing structures. The identification of structurally efficient grid-stiffened geometries also requires the integration of optimization techniques with accurate structural analyses. Optimization of composite structures has been of considerable interest in recent years. Composite plates have been optimized to maximize buckling loads,[1,2] and a summary of this work is presented in the literature survey of Ref. 2. According to Ref. 2, the basic design problem is to determine the stacking sequence of a composite laminate because, in many practical applications, the ply orientations are limited to either 0, 90 or ±45 deg, and the laminate thicknesses can only be integer multiples of commercially available ply thicknesses. Thus, the optimization of a laminate-stacking sequence involves discrete design variables associated with manufacturing constraints and it represents an integer programming problem. In Refs. 1 and 2, the genetic algorithm[3,4] was used as the optimizer, and the authors showed that the genetic algorithm is very efficient in performing an optimization that involves discrete design variables.

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The optimum design of stiffened panels that satisfies buckling constraints is of considerable importance to the aerospace industry because aerospace structures consist mostly of stiffened panels. A summary of the research work on the design optimization of stiffened panels or shells is given in Table 1 in chronological order.

Table 1 shows that researchers have examined stiffened flat panels, cylindrical panels, and circular cylindrical cylinders. Most of these references deal with axially or orthogonally stiffened panels or cylinders, with the exception of Refs. 9, 11, 12, and 18. A gradient-based optimizer has been used the most, and the genetic algorithm has been used where researchers have treated the design variables as discrete variables (e.g., Refs. 2 and 14).

Treating stiffener spacing as a design variable in the design process has been investigated by using the gradient-based optimizer (e.g., Refs. 5, 6, and 15–18), simplex method (e.g., Refs. 7 and 13), and by using a parametric study (e.g., Ref. 9). In Ref. 9, the buckling response of various grid-stiffened cylinders with different stiffener spacings and configurations are assessed using global and local analyses, and the weight of the cylinders are also computed. The performance of each cylinder is then assessed to identify the lightest-weight cylinder. Hence, the use of a parametric study is restricted to a small design space and is quite tedious.

Stiffener spacing in different directions is a discrete variable because the number of axial and transverse stiffeners is an integer number, and hence, the stiffener spacing occurs as a certain multiple of the length or width of the panel. In PANDA2,[15–18] the stiffener spacing is treated as a continuous variable and a gradient-based optimizer is used. This design philosophy is adopted because the panel dimensions may not be fixed at a preliminary design stage and the panel dimensions change as the stiffener spacing variable changes. Hence, the stiffener spacing is treated as a continuous variable for a certain number of design iterations until a near optimum is reached. Thereafter, the spacing is held constant at various levels while other design variables are permitted to vary. When this design strategy or philosophy is used, the stiffener spacing may have to be adjusted to correspond to an integer number of stiffeners before being held constant at various levels. A gradient-based method has also been used to determine the position of a fixed number of stiffeners in a panel as shown in Ref. 11. In this case, the positions of the stiffeners are continuous variables.

The simplex method[19] that has been used in Ref. 7 and 13 also assumes continuous variables, although it does not require gradient information. This simplex method will minimize a function of n variables, which depends on the comparison of function values at $(n + 1)$ vertices of a general simplex. If the objective function is highly nonlinear or contains multiple minima of comparable and/or noncomparable performance, a comparison of the objective function values at $(n + 1)$ vertices of the simplex will not be an accurate approximation of the ob-

Table 1 Summary of research work on stiffened panels or shells

Reference	Structure type	Buckling analysis	Optimization algorithm	Design variables
5 (1963)	Orthogonally stiffened flat panel	Global and local analyses	Gradient-based method	Skin thickness Stiffener height Stiffener thickness Stiffener spacings
6 (1972)	Orthogonally stiffened circular cylinder	Global and local analyses	Gradient-based method	Skin thickness Stiffener height Stiffener thickness Stiffener spacings
7 (1975)	Orthogonally stiffened circular cylinder	Global and local analyses	Simplex method	Skin thickness Stiffener height Stiffener thickness Stiffener spacings
8 (1981)	Axially stiffened panel	Complex Fourier series	Gradient-based method	Skin laminate Stiffener dimensions
9 (1985)	Grid-stiffened circular cylinder	Global and local analyses	Parametric study	Skin laminate Stiffener height Stiffener thickness Stiffener spacings Stiffening configuration
10 (1990)	Orthogrid cylindrical panel	Finite element analysis	Ranking	Skin laminate
11 (1990)	Grid-stiffened panel with symmetric stiffeners	Discrete analysis	Gradient-based method	Stiffener position Skin thickness Stiffener height Stiffener thickness
12 (1992)	Grid-stiffened circular cylinder	Finite element analysis	Gradient-based method	Skin thickness Stiffener thickness Stiffener height
13 (1992)	Orthogonally stiffened circular cylinder	Global and local analyses	Simplex method	Skin thickness Stiffener dimensions Stiffener spacings
2 (1993)	Axially stiffened panel	Complex Fourier series	Genetic	Skin laminate Stiffener laminate
14 (1995)	Axially stiffened cylindrical panel	PANDA2[a]	Genetic and response surface approximation	Skin laminate Skin laminates for Hat-stiffener, Four dimensions of Hat-stiffener
18 (1996)	Isogrid ring-stiffened cylindrical shell	PANDA2[a]	Gradient-based method	Skin thickness Stiffener thickness Stiffener height Stiffener spacings

[a]PANDA2 (Refs. 15–18).

jective function. In fact, the optimal design of stiffened panels is a search in a space with multiple minima.[2,14] Furthermore, the simplex method may provide optimal design where the stiffener spacing does not correspond to the integer number of stiffeners in the panel or shell. Hence, the simplex method is also not strictly applicable to the optimization of a stiffened panel or shell where the stiffener spacing is a design variable.

An important research work on the minimum-weight preliminary design of composite panels and shells developed, over many years at Lockheed Palo Alto Research Laboratory, which is still being developed led to the PANDA2[15–17] computer program. PANDA2 handles the optimization of ring- and/or stringer-stiffened cylindrical panels and shells. Recently capabilities for isogrid stiffening were introduced in PANDA2.[18] Stiffeners can have different cross sections, e.g., rectangular, tee, hat, and zee sections. In-plane loads, pressure, edge moment, and temperature effects can be included. Analysis is based on analytical solutions and engineering approximation. PANDA2 provides a global buckling load based on a smeared stiffener theory or on a discrete analysis, and also checks for local buckling of skin and stiffener segments. Constraints on stresses can be imposed. In addition, PANDA2 can include optimization of panels that are locally postbuckled. Also, optimization can include the effects of initial imperfections in the form of buckling modes that can be either local, global, or both. Gradient-based optimizers are used in PANDA2.

Crossley and Laananen[20] presented a genetic algorithm-based design tool for stiffened composite panels for energy absorbtion using a semiempirical approach. The number of stiffeners, stiffener shape, and stiffener lay-up are treated as design variables. Their emphasis is the crashworthiness of the structural design rather than the buckling analysis for in-plane flight loads.

The present paper presents an analysis and design strategy for grid-stiffened composite panels subjected to combined loads and a global buckling design constraint as well as local buckling constraints of skin and stiffener segments. The type of grid-stiffened panel considered in this paper has integral stiffeners and the panel is manufactured in such a way that no flange is necessary to allow for sufficient contact area for bonding. The global buckling constraint for the grid-stiffened panel is important to prevent localized skin buckling. This constraint reduces the loss of aerodynamic performance caused by the buckling of wing or fuselage surfaces and prevents the failure of a panel by stiffener-skin separation[21] after buckling. Design variables are the axial and transverse stiffener spacings, the stiffening configuration, the skin laminate, and the height and thickness of the stiffener. The genetic algorithm is used as the optimization tool for evolving the design because the stiffener spacings, stiffening configuration, stiffener thickness, and skin laminate are discrete design variables as explained earlier. The genetic algorithm used in this study is the one developed by Leriche and Haftka.[1] To the authors' knowledge, stiffener spacing was first treated as a discrete design variable in Ref. 22, which serves as the basis of this paper. Also the inclusion of the stiffening configuration, e.g., axially stiffened only, orthogonally stiffened, general grid-stiffened panel, as a design variable in the optimization of a grid-stiffened panel had not been investigated prior to Ref. 22. Hence, the main contributions of this paper include the following:

1) The development of an analysis and design tool for grid-stiffened composite panels subjected to combined in-plane loads and arbitrary boundary conditions.

2) The treatment of discrete design parameters (such as stiffener spacing, stiffening configuration, and specific lamination schemes) in a rigorous manner using the genetic algorithm.

3) The application of this design tool to generic aerospace vehicle composite structures.

Thus, an optimal design strategy for multidirectionally stiffened composite cylindrical panels using the genetic algorithm is developed where the design variables used cover most of the design aspects of grid-stiffened composite panels.

Panel Buckling Analysis

The analysis and design of grid-stiffened composite panels subjected to combined loads require several key steps. In the present study, acceptable designs are those that buckle globally and do not exhibit any local skin buckling or stiffener crippling. This design approach leads to a buckle-resistant design, and global and local analyses can be used in identifying the global buckling response of the stiffened structure.

The literature survey of past research work on stiffened panel and shells shows that global and local analyses have been widely used in the analysis and design of stiffened panels and shells. Finite element analysis has been used and although it is very accurate, it is also computationally expensive, particularly where many analyses are required for solution convergence to an optimal design. Also, when using the finite element analysis in an optimization procedure, it is very difficult to have the stiffener spacing or the stiffening configuration as design variables because of the need for remeshing of the finite element model. The discrete analysis[11] tends to be expensive when the panel or shell has a large number of stiffeners, and including the eccentricities of the stiffeners in the discrete analysis is also very difficult. A complex Fourier series analysis developed by Anderson et al.[8] and used in Ref. 2 is computationally efficient, but it is restricted to axially stiffened panel or shells only.

Researchers who have used global and local analyses have frequently neglected the anisotropic properties of the skin segments and the curvature of the skin segments in the buckling analysis. In some cases, restrictions on the aspect ratio of the rectangular skin segments and the loading are imposed to obtain closed-form solutions for the buckling of skin segments. In Ref. 9, the isogrid stiffening configuration was studied because only a buckling analysis of an orthotropic equilateral triangular plate was available. In using global and local analyses, some researchers have imposed the conditions of a simultaneous occurrence of global and local buckling (e.g., Refs. 23 and 24), and of a nonsimultaneous occurrence of global and local buckling (e.g., Refs. 6, 7, and 13). It is argued in Ref. 13 that whereas the occurrence of simultaneous global and local buckling might imply a minimum condition, it can also lead to an imperfection-sensitive design.

In the present study, an improved smeared stiffener theory[25] is used that accounts for skin-stiffener interactions and also the eccentricities of the stiffeners. The improved smeared theory is used with a Rayleigh–Ritz method presented in Ref. 26, which accounts for anisotropic material properties and different boundary conditions. Local buckling analyses of quadrilateral and triangular skin segments are done using the procedures presented in Refs. 26 and 27. These buckling analyses also account for anisotropic material properties and different boundary conditions. These solutions are in very good agreement with existing analytical solutions and finite element analysis as documented in Refs. 26 and 27. Convergence behavior and accuracy of the solution are established against known solutions and/or finite element solutions. Different shell theories have been implemented in the Rayleigh–Ritz method through tracer coefficients and is presented in detail in Ref. 28. Crippling of the stiffener segment is assessed using a method provided in Ref. 9. The nonsimultaneous occurrence of global and local buckling is used to avoid design that are imperfection sensitive.

The global buckling analysis is based on a Rayleigh–Ritz method using a first-order, shear-deformation theory and an improved smeared-stiffener modeling approach discussed in Ref. 25. The buckling analysis of local skin segments is also based on a Rayleigh–Ritz analysis using a first-order, shear-deformation theory, and it accounts for material anisotropy. Boundary restraints on the skin segments are provided by the stiffeners, and hence, the analysis must be capable of accommodating a variety of boundary conditions and skin segment shapes.[26,27] In most cases, the skin segments for grid-stiffened panels will have either a general parallelogram-shaped or a general triangular-shaped planform. In addition to analyzing the local skin segment for buckling, the local stiffener segments must be analyzed to determine whether stiffener crippling will occur.[9] Accordingly, the stiffener segment at the nodes or intersection points of the stiffeners are assumed to be clamped while the stiffener-skin attachment is assumed to be a simple support.

A summary of the steps performed as part of this optimal design strategy is given here. The first step in the design process is to assess the global buckling response of a grid-stiffened panel. Once this global buckling response is determined, the second step is to determine the local skin buckling response for the quadrilateral and/or triangular skin segments between the stiffeners. The third step is to determine whether stiffener buckling or stiffener crippling has occurred at this global buckling load level. This sequence of steps is performed repeatedly in a design cycle until an optimum or near-optimum design is obtained. These global and local analysis methods have been integrated into a computer code to provide a computationally efficient tool for predicting the buckling load of grid-stiffened composite panels.

Panel Design Procedure

The design of grid-stiffened composite panels requires that many of the design variables, such as stiffener spacing and stiffener thicknesses, may only have certain discrete values rather than varying continuously over the design space. Also, a family of acceptable designs is often needed for selecting designs that accommodate manufacturing and other constraints rather than a single-point design. Gradient-based methods for structural optimization are not appropriate in this case, because they lead to a single-point design.

Over the last several years, researchers have investigated the use of genetic algorithms as a method for evolving a given design problem to a family of near-optimum designs (e.g., see Refs. 1–4). Based on Darwin's theory of the survival of the fittest, the genetic algorithm involves the random creation of a design population that evolves toward some definition of fitness. The genetic algorithm is attractive because of the simplicity of its approach using discrete variable combinatorics. The genetic algorithm can be used directly to solve unconstrained optimization problems, while constrained optimization problems must first be transformed into an unconstrained optimization problem, e.g., use of an exterior penalty function. Stochastic processes are used to generate an initial population of individual designs, and the algorithm then applies the principles of natural selection and survival of the fittest to find improved designs as the design evolves from generation to generation. Furthermore, because the discrete design procedure works with a population of designs, it can explore a large area of design space and identify multiple minima or maxima. This attribute is a major advantage because the converged solution represents the generation that contains many optima of comparable performance in that population. The cost of having a large number of function evaluations is justified by the fact that many near-optimum designs are now available. In a gradient-based optimization procedure, only a single-point design, usually the extremum closest to the starting point, is obtained. However, different starting points can be tried to increase the chance of locating the global optimum as well as other local optima. The genetic algorithm produces a population or family of good designs, which may include the global optimal design, rather than a single design. These advantages of the genetic algorithm make it an appropriate optimizer that can be exploited in developing a design optimization tool for general grid-stiffened panels.

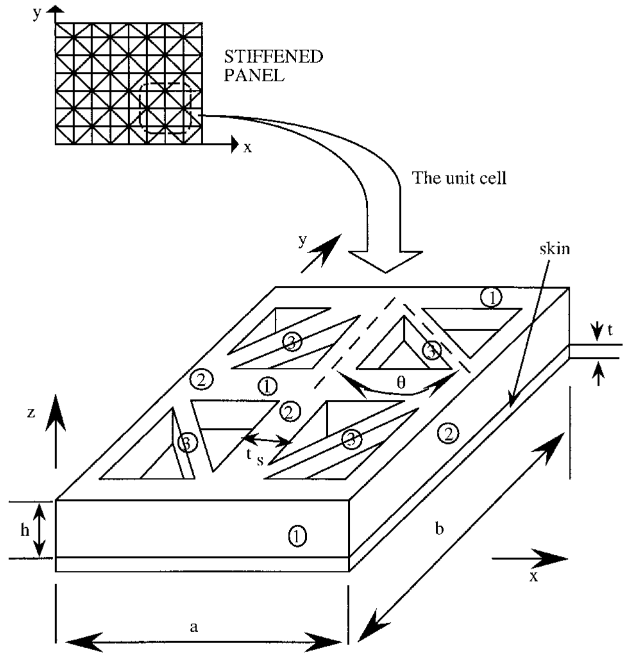

Fig. 1 Unit cell of grid-stiffened panel showing design variables: ① = axial, ② = transverse, and ③ = diagonal stiffeners.

Design Problem Definition

The present design problem is to minimize the weight per unit area of a grid-stiffened composite panel given the design loading condition, the length and width of the panel, the material properties for the skin and stiffeners, and the boundary conditions of the panel. As shown in Fig. 1, a general grid-stiffened panel may be considered as an assembly of a repetitive unit or a unit cell. A unit cell is a repetitive unit of skin and stiffener elements. The design variables include a, b, *LAMI*, *ICON*, t_s, and h. The a, b, h, and t_s are shown in the figure of the unit cell of Fig. 1. Also shown in Fig. 1 is the t, which depends on the skin-stacking sequence. The stiffening configuration depends on the combination of axial, transverse, and diagonal stiffeners in the unit cell (Fig. 1). For manufacturing and assembly reasons, all stiffeners are assumed to be of the same height and thickness. The design sought here is a panel of minimum weight in a certain design space that buckles globally at the design loads. This design problem can be defined by setting up the optimization procedures in the following way. First, the global buckling load is assumed to be a scalar multiple of the design loads and has the form

$$(N_x)_{cr} = \lambda_G N_x, \qquad (N_y)_{cr} = \lambda_G N_y, \qquad (N_{xy})_{cr} = \lambda_G N_{xy} \tag{1}$$

where N_x, N_y, and N_{xy} are the applied in-plane prebuckling loads. These values represent the design loads for the grid-stiffened panel. Second, the design constraints imposed on the panel include the following requirements:

1) The critical buckling load should be greater than or equal to the design loads, that is, $\lambda_G \geq 1$.

2) The skin segments should not buckle at the critical buckling load, that is, $\lambda_{sk} \geq 1$.

3) The stiffener segments should not cripple at the critical buckling load, that is, $\lambda_1, \lambda_2, \lambda_3 \geq 1$ where $\lambda_1, \lambda_2, \lambda_3$ are the crippling load factors of the x-direction (axial) stiffener, y-direction (transverse) stiffener, and diagonal stiffener, respectively.

The local buckling load factors of each stiffener segment type and skin segment is

$$\lambda_i = \frac{\text{critical load of local segment}}{\text{load in local segment because of } \lambda_G \times (N_x, N_y, N_{xy})} \tag{2}$$

where $i = 1, 2, 3$, and sk, and the general form of each constraint equation is written as

$$g_j = [(1/\lambda_j) - 1] \leq 0.0 \quad j = 1, \ldots, N_c \tag{3}$$

Finally, because the minimum panel weight is sought, the fitness measure is related to the reciprocal of the panel weight with the buckling constraints included. Hence the fitness expression based on an exterior penalty function approach is

$$\text{fitness} = \left[\frac{Q}{F(\boldsymbol{X}, r_i)}\right] = \max \frac{Q}{W(\boldsymbol{X}) + r_i \sum_{j}^{N_c} [|g_j(\boldsymbol{X})| + g_j(\boldsymbol{X})]^2} \tag{4}$$

The value of the exterior penalty function is zero when there is no constraint violation. When there is a slight constraint violation, the value of the exterior penalty function is very small. For a severe violation, the value of the penalty function is large, and it is very large for very severe constraint violation. Therefore, this exterior quadratic penalty function approach allows a design with a slight constraint violation to retain a good fitness compared to a heavy design without a constraint violation. A design with a severe constraint violation is allowed a very poor fitness. Hence the motivation for using the exterior quadratic penalty function. Once the global buckling load factor has been determined, the loads acting on the stiffener and skin segments have to be determined by distributing the loads among the skin and stiffeners based on their extensional stiffnesses. The procedure for distributing the applied loads for a general grid-stiffened panel is discussed in Refs. 11 and 28.

The weight per unit area of the grid-stiffened panel is

$$W = (\rho/ab)(w_1 + w_2 + w_3 + w_s) \tag{5}$$

Design Process Based on a Genetic Algorithm

Implementation of the design process based on a genetic algorithm is shown schematically in Fig. 2. The genetic algorithm used is the one developed by Leriche and Haftka,[1] where the binary coding is not used to code the design variables. Integers are assigned to each discrete value of each design variable and these integers are used to encode each design. As pointed out in Ref. 1, the optimal population size as derived by Goldberg[3] does not account for operators like permutation and mutation and thus overestimate the optimal population size. When using this implementation of the algorithm, a realistic population size is found to be

$$M \geq 2 \times N_d \tag{6}$$

In this paper, $M = 8$ when N_d is 3, and $M = 12$ when N_d is 6. These values for the population size are found to work very well for the problems considered. The selection scheme is based on the probabilities of crossover, mutation, and permutation. Note that Goldberg's genetic algorithm[3] has only crossover as a selection scheme. In this paper the probabilities used are 1.0, 0.10, and 0.95 for crossover, mutation, and permutation, respectively. These probabilities were selected from the studies conducted in Ref. 1, where the following facts were found:

1) The optimum probability for crossover is 1.0 for unconstraint as well as constraint optimization.

2) The optimum probability for mutation is 0.01 for unconstraint optimization and 0.10 for constraint optimization.

3) The optimum probability of permutation should be much higher than that of mutation for constraint optimization.

The design process begins with a random selection of a specified number of designs that comprises the initial population, i.e., first generation, for the genetic algorithm. The problem parameters such as material properties, panel length and width, boundary conditions of the panel, and design loads are input into the analysis processor routine. The buckling analyses are performed that provide the critical eigenvalues for the global buckling response of the grid-stiffened panel, and the local buckling response of the skin and stiffener segments. The weight per unit area of the grid-stiffened panel is also computed. This procedure is repeated for each design configuration in the population. The fitness processor then evaluates the fitness of each design using Eq. (4), and assigns a rank based on the fitness expression or the objective function. The current population of design configurations is then processed by the genetic operators (crossover, mutation, and permutation) to create a new population of design configurations for the subsequent generations that combines the most desirable characteristics of the previous generations. Designs from previous generations may be replaced by new ones, i.e., children, except for the most fit designs, i.e., parents, which are always included in the next generation. The process is repeated until design convergence is obtained. Convergence is defined herein by specifying a maximum number of generations (*NSTOP*) that may occur without any improvement in the most fit design. The design procedure is demonstrated on flat and curved grid-stiffened panels in the following sections.

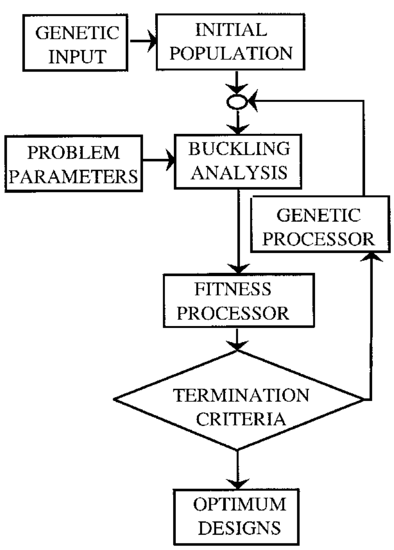

Fig. 2 Flow chart showing design optimization procedure using the genetic algorithm.

Numerical Results for Flat Grid-Stiffened Panels

A 20.0-in.-long and 56.0-in.-wide flat grid-stiffened composite panel representative of a generic transport helicopter fuselage structural component is designed to demonstrate the capabilities of the present design optimization tool. The panel is optimized for a load case with 400.0 lb/in. of axial compression, which is considered to be the critical load case for this panel. The termination criterion (*NSTOP*) is 30 genera-

Table 2 Design space for flat grid-stiffened panels

a_s, b_s, in.	h, in.	t_s, in.
6.667, 11.200	0.49375	0.060
5.714, 10.182	0.50000	0.066
5.000, 8.615	0.50625	0.072
4.444, 8.000	0.51250	0.078
4.444, 7.467	0.51875	0.084
4.000, 7.000	0.52500	0.090
3.636, 6.222	0.53125	0.096
3.333, 5.894	0.53750	0.102

Table 3 Best designs obtained by the genetic algorithm for a grid-stiffened panel

Design variables, in.				Buckling load factors				Weight, lb/ft^2
a_s	b_s	h	t_s	λ_G	λ_{sk}	λ_1	λ_3	
5.000	8.615	0.5375	0.06	0.995	1.024	1.341	21.208	0.578
4.444	8.000	0.5375	0.06	1.047	1.179	1.470	25.608	0.596
4.444	7.467	0.5125	0.06	0.991	1.392	1.637	24.507	0.594
4.444	7.467	0.5125	0.072	1.139	1.285	2.146	32.120	0.634
4.444	8.000	0.5375	0.072	1.207	1.084	1.920	33.458	0.636

Table 4 Buckling loads for grid-stiffened panels

Load case number	N_x, lb/in.	N_{xy}, lb/in.	Critical buckling load factor, λ
1	400.0	0.000	0.995[a]
2	396.0	130.0	1.002[a]
3	326.0	178.0	1.211[a]
4	271.0	139.0	1.458[a]
5	174.0	154.0	2.206[b]

[a]Global buckling. [b]Skin buckling.

tions, and the population size is set to eight. The probabilities used for crossover, mutation, and permutation are 1.0, 0.10, and 0.95, respectively. Also, the penalty parameter r_i is kept constant for all iterations because the genetic algorithms maximize Eq. (4) more efficiently. The skin laminate stacking sequence selected is $[\pm 45/90/0]_s$ with each ply thickness being 0.006 in. The stiffeners are made of 0-deg material only. The nominal ply mechanical properties used are E_{11} = 20.2 Msi, E_{22} = 1.9 Msi, $G_{12} = G_{13} = G_{23}$ = 0.73 Msi, and ν_{12} = 0.3. The ρ is 0.0570 lb/in.[3] The grid-stiffened panel has simply supported boundary conditions on all edges, and the skin segments are also considered to be simply supported. The grid-stiffened panel is assumed to have only axial and diagonal stiffeners. The axial and transverse stiffener spacings considered are such that all stiffener patterns closely approximate an isogrid configuration (angle θ in Fig. 1 is approximately 30 deg). Hence, a and b are not independent, but are considered as a single design variable, i.e., (a_s, b_s) is one design variable. h and t_s are also design variables. The design space explored is indicated in Table 2, where the height of the triangle $(b_s/2)$ is kept between 2.9 and 6.0 in., and the stiffener aspect ratio (h/t_s) is kept between 4.5 and 9.0 because of manufacturing constraints. Each design variable is permitted to assume only eight discrete values for the particular implementation of the genetic algorithm being used—this is not a limitation of the genetic algorithm in general.

The five best designs from the optimization are shown in Table 3. These designs buckle globally at the indicated load factor of λ_G because the local buckling load factors (λ_{sk}, λ_1, and λ_3) are all greater than 1. The first and third designs have a global buckling load factor λ_G of 0.995 and 0.991, respectively, and could still represent acceptable designs. The second and third designs have different stiffener spacing, and yet the global buckling load factor and weight per unit area of these two panels are very close to one another. The fourth and fifth designs are also comparable in performance. Hence, the ability of the genetic algorithm to obtain multiple optima of comparable performance is demonstrated. The best design for the grid-stiffened panel is the first design given in Table 3. Design convergence was achieved after 40 iterations.

The grid-stiffened panel is now assessed using representative flight-load conditions given in Table 4. The results are also shown in Table 4. The panel buckles globally for the load cases considered except the load case of N_x = 174.0 lb/in. and N_{xy} = 154.0 lb/in. The λ_G for this load case is 2.2382, and the λ_{sk} is 0.9855. The buckling load factor is 2.2057 for the load case of N_x = 174.0 lb/in. and N_{xy} = 154.0 lb/in., because $\lambda_{sk} < 1.0$, the skin segment buckles at a load factor of $\lambda_{sk} \times \lambda_G$ = 2.2057. Hence, the panel does not buckle at a load case of N_x = 174.0 lb/in. and N_{xy} = 154.0 lb/in.

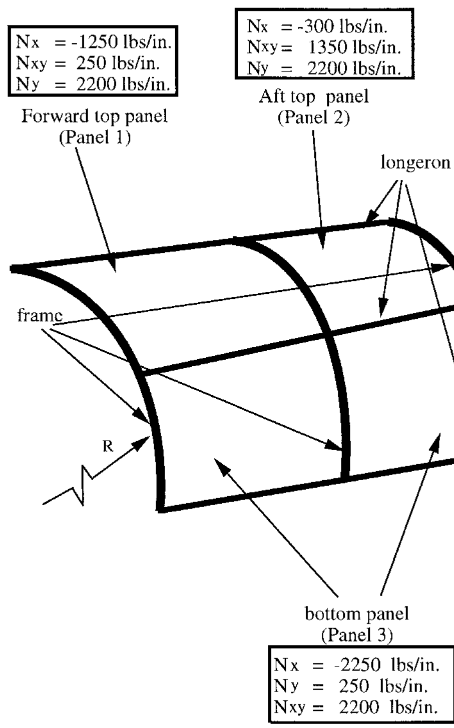

Fig. 3 Side quadrant panel of a fuselage structure.

Numerical Results for Curved Grid-Stiffened Panels

The fuselage design of a generic wide-body transport aircraft is typically divided into four different quadrants. These quadrants include a crown panel, two side panels and a keel panel. A side-quadrant panel is considered herein and designed for global buckling. The side-quadrant panel is shown in Fig. 3 and longerons and frames divide the side-quadrant panel into four curved panels. Each panel is chosen to be 22.0-in. long and 22.0-in. wide with a radius of 120.0 in. in the width direction. The dimension of 22.0 in. corresponds to the frame spacing. Panel 1 is the forward–top panel of the side-quadrant panel and is subjected to N_x = 1250 lb/in., N_{xy} = 250 lb/in., and N_y = −2200 lb/in. (hoop tension). Panel 2 is the aft–top panel of the side-quadrant panel and is subjected to N_x = 300 lb/in., N_{xy} = 1350 lb/in., and N_y = −2200 lb/in. (hoop tension). Panel 3 is the bottom panel of the side-quadrant panel and is subjected to N_x = 2250 lb/in., N_{xy} = 250 lb/in., and N_y = −2200 lb/in. (hoop tension). The panel hoop tension is a result of internal pressurization of the fuselage. The nominal ply mechanical properties used are E_{11} = 20.2 Msi, E_{22} = 1.9 Msi, $G_{12} = G_{13} = G_{23}$ = 0.73 Msi, and ν_{12} = 0.3. The ρ is 0.0570 lb/in.[3] The grid-stiffened panel is assumed to have simply supported boundary conditions on all edges, and the individual skin segments are also considered to be simply supported. The stiffeners are made of unidirectional material. The termination criterion (*NSTOP*) for the design evolution is 25 generations with no improvement in the most fit design, and the population size

Table 5 Design space for a, b, h, and t_s of curved panels 1 and 3

a, in.	b, in.	h, in.	t_s, in.
11.000	11.000	0.49375	0.060
8.800	8.800	0.50000	0.066
7.333	7.333	0.50625	0.072
6.286	6.286	0.51250	0.078
5.500	5.500	0.51875	0.084
4.889	4.889	0.52500	0.090
4.400	4.400	0.53125	0.096
4.000	4.000	0.53750	0.102

Table 6 Design space for a, b, h, and t_s of curved panel 2

a, in.	b, in.	h, in.	t_s, in.
11.000	11.000	0.30	0.042
8.800	8.800	0.32	0.048
7.333	7.333	0.34	0.054
6.286	6.286	0.36	0.060
5.500	5.500	0.38	0.066
4.889	4.889	0.40	0.072
4.400	4.400	0.42	0.078
4.000	4.000	0.44	0.084

is set to 12. The probabilities used for crossover, mutation, and permutation are 1.0, 0.10, and 0.95, respectively. Also, r_i is kept constant for all iterations.

The design variables of the grid-stiffened panel shown in Fig. 1 are a, b, h, t_s, *LAMI*, and *ICON*, which is a design variable indicating the combination of axial, transverse, and diagonal stiffeners in a unit cell. The design space explored for a, b, h, and t_s is shown in Table 5 for panels 1 and 3, and in Table 6 for panel 2. The design space for *LAMI* and *ICON* is described in Table 7 for panels 1, 2, and 3, e.g., when *LAMI* = 1, then the skin-stacking sequence is $[\pm 45/0]_{2s}$ and when *ICON* = 1, then the stiffening configuration consists of axial stiffeners. Each ply thickness for the skin laminate is 0.006 in. The minimum skin thickness is 0.072 in., and the ply orientation is restricted to 0, 90, and ±45 deg. These orientations are commonly used in industry, wherein each laminate has a ±45-deg ply on the outer surfaces to provide a scratch resistant surface. Hence, the laminates in Table 7 are representative of laminates from manufacturing and other constraints. Other design variables could be introduced for the stacking sequence; however, only commonly used laminates are considered herein. In either design space, the minimum stiffener spacing is restricted to 4 in., and the aspect ratio of the stiffener h/t_s is kept between 3.5 and 10.5 because of manufacturing constraints. Each design variable can assume eight discrete values. A modified Sanders–Koiter shell theory is used to account for transverse shear deformation in the Rayleigh–Ritz buckling analysis.[28]

The results obtained for panel 2 using the present optimization tool are shown in Table 8. The panel designs presented in Table 8 buckle globally at the corresponding global load factor of λ_G. The genetic algorithm produces a large pool of acceptable designs in this case. Most of the acceptable designs only have axial stiffeners. These axially stiffened panels have stiffener spacings similar to those of the first three designs presented in Table 8, but with different values of stiffener height and thickness. The fourth design in Table 8 has transverse and diagonal stiffeners (*ICON* = 6); however, this design is 14% heavier than the first design. Note that the fourth design in Table 8 has $\lambda_3 = -0.3$, which means the diagonal stiffener is in tension and does not buckle even though $|\lambda_3| < 1.0$. Panels stiffened in multiple directions have redundant load paths and typically exhibit better damage tolerance characteristics than panels stiffened in one direction. Therefore, the design process was repeated using a modified design space for the stiffening configuration where values of *ICON* = 1 and 2 are replaced by *ICON* = 5 and 6, respectively. That is, all possible design configurations include stiffeners in multiple directions. The results for this optimization are shown in Table 9. The panels presented in Table 9 buckle globally at the corresponding λ_G. The genetic algorithm produces a large pool of acceptable designs with axial and transverse stiffeners (*ICON* = 3). These panels have stiffener spacings represented by the first four designs of Table 9, with variation in stiffener height and thickness and with a skin laminate stacking sequence of $[\pm 45/0]_{2s}$ (*LAMI* = 1) or $[\pm 45/90]_{2s}$ (*LAMI* = 2). The weight of each panel is comparable to the weight of the axially stiffened panels presented in Table 8. The fifth design has both axial and diagonal stiffeners (*ICON* = 5), while the sixth design is the same as the fourth design presented in Table 8. The best design for panel 2 is the design with transverse and diagonal stiffeners, which is the fourth design in Table 8 and the sixth design in Table 9. This design is preferred over the lighter-weight axially stiffened panel and the axially and transversely stiffened panel because the panel with diagonal stiffening may be more damage tolerant.[21] The sixth design in Table 9 is also preferred over the fifth design in Table 9, because it has more stiffeners than the axially and diagonally stiffened panel and its weight is close to that of the latter. The convergence behavior of the genetic algorithm for panel 2 is shown in Fig. 4. The convergence is faster when there is no restriction on the stiffening configuration variable (*ICON*). The optimization process is performed for panels 1 and 3, and the

Table 7 Description of design space for *ICON* and *LAMI* for panels 1, 2, and 3

Integer value	*LAMI* ply thickness = 0.006-in.	*ICON*
1	$[\pm 45/0]_{2s}$	Axial stiffeners
2	$[\pm 45/90]_{2s}$	Transverse stiffeners
3	$[\pm 45/0/90]_{2s}$	Axial and transverse stiffeners
4	$[\pm 45/0_2]_{2s}$	Diagonal stiffeners
5	$[\pm 45/90_2]_{2s}$	Axial and diagonal stiffeners
6	$[\pm 45/0_2/90]_{2s}$	Transverse and diagonal stiffeners
7	$[\pm 45/0/90_2]_{2s}$	Axial, transverse and diagonal stiffeners
8	$[45/0_2/90]_{2s}$	No stiffeners

Table 8 Best designs obtained by the genetic algorithm for grid-stiffened curved panel 2

Design variables	Buckling load factors	Weight, lb/ft^2
b = 11.0 in.	$\lambda_G = 1.061$	0.6232
h = 0.360 in.	$\lambda_{sk} = 16.494$	
t_s = 0.060 in.	$\lambda_1 = 2.745$	
LAMI = $[\pm 45/0]_{2s}$		
ICON = 1		
b = 7.333 in.	$\lambda_G = 0.998$	0.6229
h = 0.340 in.	$\lambda_{SK} = 221.17$	
t_s = 0.042 in.	$\lambda_1 = 1.62$	
LAMI = $[\pm 45/0]_{2s}$		
ICON = 1		
b = 6.2857 in.	$\lambda_G = 1.129$	0.6283
h = 0.340 in.	$\lambda_{sk} = 206.79$	
t_s = 0.042 in.	$\lambda_1 = 1.45$	
LAMI = $[\pm 45/0]_{2s}$		
ICON = 1		
a = 5.5 in.	$\lambda_G = 0.999$	0.711
b = 6.2857 in.	$\lambda_{sk} = 5.01$	
h = 0.320 in.	$\lambda_2 = -.30$[a]	
t_s = 0.054 in.	$\lambda_3 = 1.650$	
LAMI = $[\pm 45/0]_{2s}$		
ICON = 6		

[a]Stiffener segment is in tension.

Table 9 Best designs obtained by the genetic algorithm for grid-stiffened curved panel 2, with *ICON* = 1, 2 excluded from the design space

Design variables	Buckling load factors	Weight, lb/ft^2
a = 11.0 in.	λ_G = 1.038	0.6389
b = 8.8 in.	λ_{sk} = 3.05	
h = 0.340 in.	λ_1 = 2.01	
t_s = 0.042 in.	λ_2 = −0.149[a]	
$LAMI$ = $[\pm45/0]_{2s}$		
$ICON$ = 3		
a = 11.0 in.	λ_G = 1.022	0.6396
b = 11.0 in.	λ_{sk} = 2.07	
h = 0.340 in.	λ_1 = 1.28	
t_s = 0.048 in.	λ_2 = −0.360[a]	
$LAMI$ = $[\pm45/90]_{2s}$		
$ICON$ = 3		
a = 11.0 in.	λ_G = 1.076	0.6411
b = 7.333 in.	λ_{sk} = 4.141	
h = 0.320 in.	λ_1 = 1.07	
t_s = 0.042 in.	λ_2 = −0.366[a]	
$LAMI$ = $[\pm45/90]_{2s}$		
$ICON$ = 3		
a = 8.8 in.	λ_G = 1.083	0.6442
b = 8.8 in.	λ_{sk} = 2.979	
h = 0.340 in.	λ_1 = 1.08	
t_s = 0.042 in.	λ_2 = −0.300[a]	
$LAMI$ = $[\pm45/90]_{2s}$		
$ICON$ = 3		
a = 11.0 in.	λ_G = 2.160	0.6938
b = 11.0 in.	λ_{sk} = 2.442	
h = 0.340 in.	λ_1 = 3.23	
t_s = 0.084 in.	λ_3 = 1.234	
$LAMI$ = $[\pm45/0]_{2s}$		
$ICON$ = 5		
a = 5.5 in.	λ_G = 0.999	0.711
b = 6.2857 in.	λ_{sk} = 5.01	
h = 0.320 in.	λ_2 = −0.30[a]	
t_s = 0.054 in.	λ_3 = 1.650	
$LAMI$ = $[\pm45/0]_{2s}$		
$ICON$ = 6		

[a]Stiffener segment is in tension.

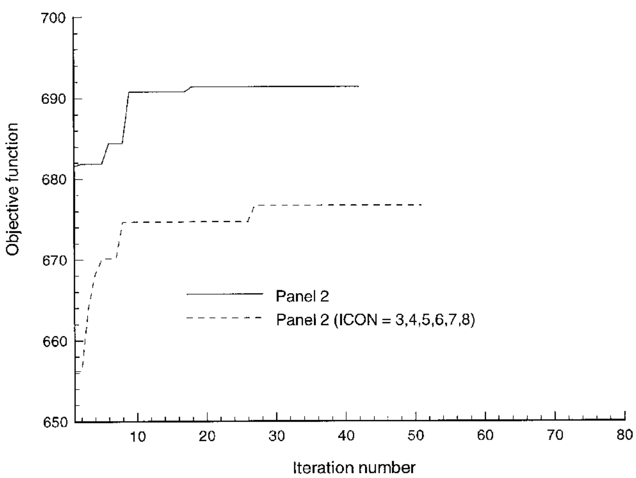

Fig. 4 Design convergence history for a composite grid-stiffened curved panel (panel 2).

best design obtained for these two panels are shown in Table 10. The optimum designs in Tables 8–10 have *LAMI* = 1 or 2 as calculated by the genetic algorithm, and the laminates are those with the fewest plies. This is not surprising because the weight of the skin contributes more to the weight of a stiffened panel than does the weight of the stiffeners for a stiffened panel where the aspect ratio of the stiffener (h/t) is between 4 and 10. Hence, optimum designs of such panels will always occur at the minimum skin thickness such that the local buckling constraint on the skin segment is not violated. The local buckling of the skin segment depends on the stiffener spacing, the stiffening configuration, the in-plane loading, the geometry of the skin segment, the skin laminate, and the size and stiffness of the stiffener.

Table 10 Best designs obtained by the genetic algorithm for grid-stiffened curved panels 1 and 3

Design variables	Weight, lb/ft^2	Buckling load factors
	Panel 1	
a = 4.8890 in.	0.819	λ_G = 1.080
b = 6.2857 in.		λ_{sk} = 1.290
h = 0.500 in.		λ_2 = −0.394[a]
t_s = 0.060 in.		λ_3 = 18.189
$LAMI$ = $[\pm45/90]_{2s}$		
$ICON$ = 6		
	Panel 3	
a = 5.500 in.	0.874	λ_G = 1.014
b = 4.400 in.		λ_{sk} = 1.369
h = 0.50625 in.		λ_2 = −0.781[a]
t_s = 0.072 in.		λ_3 = 1.191
$LAMI$ = $[\pm45/90]_{2s}$		
$ICON$ = 6		

[a]Stiffener segment is in tension.

Table 11 Buckling loads for the best design of panel 3 subjected to the load cases of panels 1 and 2

Panel number	Load, lb/in.	Buckling load factors
1[a]	N_x = 1250	λ_G = 2.114
	N_y = −2200	λ_{sk} = 1.488
	N_{xy} = 250	λ_2 = −0.374[b]
		λ_3 = 1.0113
2[c]	N_x = 300	λ_G = 4.595
	N_y = −2200	λ_{sk} = 2.409
	N_{xy} = 1350	λ_2 = −0.172[b]
		λ_3 = 0.255

[a]Global buckling at λ_G = 2.114.
[b]Stiffener segment is in tension.
[c]Local buckling of diagonal stiffener at $\lambda_{critical}$ = 1.172.

The results for the best design obtained for panel 3 for the design load cases used for panels 1 and 2 are shown in Table 11. For the design loads corresponding to panel 1, the panel buckles globally at a buckling load factor of 2.114, and hence, this design represents a conservative design for these loads. For the design loads corresponding to panel 2, the buckling load factor of the diagonal stiffener is 0.255, and, hence, the buckling deformation contains local buckling of the diagonal stiffener at a load factor of $\lambda_3 \times \lambda_G = 1.172$. Therefore, this design also represents a conservative design for the design loads for panel 3. Hence, the best design for panel 3 can also be used for panels 1 and 2 with a weight penalty of 6.7 and 22.9%, respectively, when compared to their respective best design. For a grid-stiffened fuselage structure construction, it is desirable that all the panels have the same stiffener pattern because of both manufacturing and structural considerations. The region where stiffeners from two different panels connect, i.e., different fuselage quadrants indicated by heavy lines in Fig. 3, have to be joined. This area is a critical area, particularly if the stiffening patterns of adjacent quadrants are not the same. To alleviate that problem, these regions have to be reinforced, which could result in a higher weight.

Concluding Remarks

A minimum-weight design optimization tool with buckling constraints has been developed for grid-stiffened panels using global and local buckling analyses and a genetic algorithm. Design variables used are axial and transverse stiffener spacing, stiffener height and thickness, skin-laminate stacking sequence, and stiffening configuration. Results for flat and curved grid-stiffened panels indicate that the best design configurations obtained by the genetic algorithm depend mostly on the load cases. For most combined applied load cases, the best designs have diagonal stiffeners. However in certain cases, the best design obtained by the genetic algorithm, based on buckling considerations only, may not be suitable for structural applications because of manufacturing, joining, and damage-tolerance considerations. In this case, the pool of acceptable designs obtained by the genetic algorithm provides alternative design options for a given structural application.

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